Math 126 Summer 2026 HW 2

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July 3, 2026

Problem 1: (Borthwick 4.1, modified) Suppose $u(t,x)$ is a solution to the wave equation

$$\partial_t^2 u - c\partial_x^2 u = 0$$

Part A.) Assume $u(0,x)$, $u(t/2, x + c\frac{t}{2})$ and $u(t/2, x - c\frac{t}{2})$ are known values. Write a formula for $u(t,x)$ given these three values (Hint: Draw a picture, and consider the domain of dependence of each point).

Part B.) Assume that $v(t,x)$ satisfies

$$\partial_t^2 v - \partial_x^2 v = 1$$

and $v(0,2) = 1$, $v(1,3) = v(1,1) = 2$. What is $v(2,2)$?

Problem 2: Show that if $u(t,x)$ satisfies

$$\partial_t^2 u - 2\partial_t u - 2\partial_x u - \partial_x^2 u = 0$$

then $w(t,x) = e^{x-t}u$ satisfies the wave equation. Find a formula for u give initial conditions $(u, u_t)|_{t=0} = (g, h)$.

Problem 3: (Borthwick 4.7) Let $u(t,x)$ solve the Schrödinger equation

$$\begin{cases} \partial_t u - i\Delta u = 0 \\ u(0,x) = g(x) \end{cases}$$

for $g(x) \in C_c^\infty(\mathbb{R})$.

Part A.) Show that $\int_{\mathbb{R}^n} |u|^2 dx = \int_{\mathbb{R}^n} |g|^2 dx$ (assume that both integrals are defined, and that the Leibniz rule and Green's identities are justified).

Part B.) Show the solution u is uniquely determined by the initial condition g.

Problem 4:

Consider the equation

$$t\partial_t u - (x+1)\partial_x u = 0$$

Part A.) Use the method of separation of variables to form a family of solutions.

Part B.) Given initial condition $u(1,x) = x$, solve the equation using the method of characteristics. Are the two solutions the same?

Part B.)